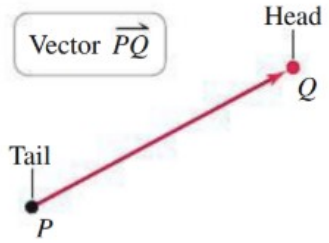
Section 13.1

Vectors in $\mathbb{R}^2$

We have two interpretations of vectors in the plane, both conveying the concept of a vector as a way to store a magnitude and a direction simultaneously.
The graphical:



And via coordinates or **components**: $\vec{v} = \langle x_0, y_0 \rangle$. The components x_0 and y_0 correspond to amounts of change in the x and y directions in the usual Cartesian coordinate system. The points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the figure above give the coordinate vector $\vec{PQ} = \langle x_2 - x_1, y_2 - y_1 \rangle$.

Definition 13.1.1. Scalar Multiples and Parallel Vectors
Given a scalar c and a vector $\vec{v}$, the **scalar multiple** $c\vec{v}$ is a vector whose length is $|c|$ multiplied by the length of $\vec{v}$. If $c > 0$, the $c\vec{v}$ has the same direction as $\vec{v}$. If $c < 0$, the $c\vec{v}$ and $\vec{v}$ point in opposite directions. Two vectors are **parallel** if they are scalar multiples of each other.

In terms of components, scalar multiplication works by scaling each individual component

$$c\vec{v} = c\langle x_0, y_0 \rangle = \langle cx_0, cy_0 \rangle$$

To compute the magnitude of a vector we use the Pythagorean Theorem:

Definition 13.1.2. Magnitude of a Vector
Given the points $P(x_1, y_1)$ and $Q(x_2, y_2)$, the **magnitude** of $\vec{PQ}$ denoted $|\vec{PQ}|$ is the distance between P and Q.

$$|\vec{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The mangitude of $\vec{v} = \langle x_0, y_0 \rangle$ is $|\vec{v}| = \sqrt{x_0^2 + y_0^2}$

Definition 13.1.3. Vector Operations
Vector addition and scalar multiplication is component wise. Suppose c is a scalar and $\vec{u} = \langle u_1, u_2 \rangle, \vec{v} = \langle v_1, v_2 \rangle$.

$$\begin{aligned} \vec{u} + \vec{v} &= \langle u_1 + v_1, u_2 + v_2 \rangle \\ \vec{u}u - \vec{v} &= \langle u_1 - v_1, u_2 - v_2 \rangle \\ c\vec{u} &= \langle cu_2, cu_2 \rangle \end{aligned}$$

Example 13.1.4. Let $\vec{u} = \langle -1, 2 \rangle, \vec{v} = \langle 2, 3 \rangle$.

1. Evaluate $|\vec{u} + \vec{v}|$.

2. Write the component vector of $2\vec{u} - 3\vec{v}$.

3. Find two vectors half as long as $\vec{u}$ and parallel to $\vec{u}$.

A **unit vector** is any vector of magnitude 1. Two important unit vectors are the coordinate unit vector $\vec{\imath} = \langle 1, 0\rangle, \vec{\jmath} = \langle 0, 1\rangle$. These vectors are directed along the positive x and y-axes. We can write any vector in $\mathbb{R}^2$ as a sum of the unit vectors $\vec{\imath}$ and $\vec{\jmath}$.

$$\begin{aligned}\vec{v} &= \langle 3, 4\rangle \\ &= 3\vec{\imath} + 4\vec{\jmath}\end{aligned}$$

Question 13.1.5. How to construct a unit vector in the same direction as a given vector $\vec{v}$?

Summary: Properties of Vector Operations
Suppose $\vec{u}, \vec{v}, \vec{w}$ are vectors, a and c are scalars.

1. $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
2. $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
3. $\vec{v} + \vec{0} = \vec{v}$
4. $\vec{v} + (-\vec{v}) = \vec{v} - \vec{v} = \vec{0}$
5. $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$
6. $(a + c)\vec{v} = a\vec{v} + c\vec{v}$
7. $0\vec{v} = \vec{0}$
8. $c\vec{0} = \vec{0}$
9. $1\vec{v} = \vec{v}$
10. $a(c\vec{v}) = (ac)\vec{v}$

Applications

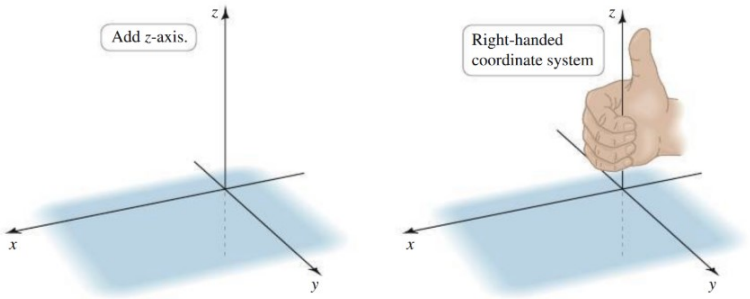
Example 13.1.6. Suppose the water in a river moves southwest at 4 mi/hr and a motorboat travels due east at 15 mi/hr relative to the shore. Determine the speed of the boat and its heading relative to the moving water.

Example 13.1.7. A 400-lb weight is suspended from two chains that form 60 degree angles with a horizontal ceiling. How much weight does each chain support?

Section 13.2

xyz-Coordinate System

We create three-dimensional space $\mathbb{R}^3$ by adding a new coordinate axis, the z-axis to $\mathbb{R}^2$. To determine what directions the positive axes will be in, we use the **right-handed coordinate system** for our convention.



With these coordinates axes established we can represent a point in $\mathbb{R}^3$ with coordinates $P(x_0, y_0, z_0)$.

Simple Planes

The xy-plane in $\mathbb{R}^3$ is the set of points that have a z-coordinate of 0: $\{(x, y, z)|z = 0\}$ or simply represented by the equation $z = 0$.

Example 13.2.8. 1. Describe the yz-plane.

2. Describe the plane parallel to the xz-plane that intersects the point $(2, 3, -1)$.

Distances

Given two points $P(x_0, y_0, z_0), Q(x_1, y_1, z_1)$ in $\mathbb{R}^3$ we define the distance between P and Q to be the length of the straight line segment between them. To measure this length we use the Pythagorean Theorem similar to how we measured distance in $\mathbb{R}^2$.

A **sphere** S is the set of all points that are at a fixed distance r from a central point P. r is the **radius** of the sphere, P its **center**. A **ball** B is the set of all points at most distance r from a central point P. We can describe both spheres and balls algebraically:

Example 13.2.9. Describe the set of points that satisfy the equation $x^2 + y^2 + z^2 - 2x + 6y - 8z = -1$.

Vectors in $\mathbb{R}^3$

Vectors in $\mathbb{R}^3$ are the natural extensions of vectors in the xy-plane: we simply introduce a third component (they are also a little harder to draw by hand). The component vector $\vec{v} = \langle x_0, y_0, z_0 \rangle$ has its tail at the origin and head at the point $P(x_0, y_0, z_0)$.

Definition 13.2.10. Vector Operations in $\mathbb{R}^3$
Vector addition and scalar multiplication is component wise. Suppose c is a scalar and $\vec{u} = \langle u_1, u_2, u_3 \rangle, \vec{v} = \langle v_1, v_2, v_3 \rangle$.

$$\begin{aligned} \vec{u} + \vec{v} &= \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle \\ \vec{u}u - \vec{v} &= \langle u_1 - v_1, u_2 - v_2, u_3 - v_3 \rangle \\ c\vec{u} &= \langle cu_2, cu_2, cu_3 \rangle \end{aligned}$$

Definition 13.2.11. Magnitude of a Vector
Given the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$, the **magnitude** of $\vec{PQ}$ denoted $|\vec{PQ}|$ is the distance between P and Q.

$$|\vec{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The mangitude of $\vec{v} = \langle x_0, y_0, z_0 \rangle$ is $|\vec{v}| = \sqrt{x_0^2 + y_0^2 + z_0^2}$